Review for Quiz on Sequences/Series

Name ____________

1) Write out the first four terms of the series & then find the sum.

a. $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right)$

b. $\sum_{n=0}^{\infty}(-1)^n\frac{5}{4^n}$

2) Determine if $\sum_{n=1}^{\infty}\frac{1}{\left(\sqrt{5}-1\right)^n}$ converges or diverges.

3) Find the formula for the n^{th} term of the sequence given by:

a) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9} \ldots$

b) $1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9} \ldots$

4) Determine the convergence or divergence of $a_n = (-1)^n \frac{n-1}{n+1}$. If the sequence converges, find its limit.

5) Find the first six terms and the 50^{th} term for the sequence: $d_n = n^2 - 2n$.

6) Find the first four terms and the 8^{th} term for the sequence: $u_1 = 1; u_2 = 2;\ u_n = u_{n-1} + u_{n-2}$ for all $n \geq 3$.

7) Given the infinite series $\sum_{n=1}^{\infty}(-1)^n$, find the sequence of partial sums $S_1, S_2, S_3, S_4,$ and S_5.

8) Find the sum of the series: $\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}$

9) Determine if each series converges or diverges. If it converges, find the sum.

a) $\frac{1}{17}+\frac{1}{289}+\frac{1}{4913}+\ldots+\frac{1}{17^k}+\ldots$

b) $1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\ldots+\left(\frac{1}{2}\right)^n+\ldots$

10) If the infinite series $\sum_{n=1}^{\infty} a_n$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$, what is the sum of the series?

11) **Find the value of each infinite series.**

a) $\sum_{n=1}^{\infty} -\frac{7}{(-3)^n}$

b) $\sum_{n=0}^{\infty}\frac{1}{3^n}$

12) The nth-Term Test can be used to determine divergence for which of the following series?

I. $\sum_{n=1}^{\infty} \sin 2n$ II. $\sum_{n=1}^{\infty}\left(2+\frac{3}{n}\right)$ III. $\sum_{n=1}^{\infty}\frac{n^3+1}{n^2}$

(A) II only (B) III only (C) I and II only (D) I, II, and III

13 **For each of the following series, determine the convergence or divergence of the given series. State the reasoning behind your answer.**

a) $\sum_{n=1}^{\infty} \frac{3-2n}{5n+1}$

b) $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

14 **Find the interval of convergence for each power series.**

a) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{4^n}$

b) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$

15 The Maclaurin series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!}$ represents which function $f(x)$

(A) $\sin x$

(B) $-\sin x$

(C) $\frac{1}{2}(e^x - e^{-x})$

(D) $e^x - e^{-x}$

16 Find the second-degree Taylor Polynomial for the function $f(x) = \frac{\cos x}{1-x}$ about $x = 0$.

17 What is the coefficient of x^2 in the Maclaurin series for the function $f(x) = \left(\frac{1}{1+x}\right)^2$?

18) For $x > 0$, the power series defined by $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!}$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$

19) Find the Maclaurin Series for the function $f(x) = e^{-3x}$. Write the first four non-zero terms.

20) What is the Taylor series expansion about $x = 0$ for the function $f(x) = \frac{\sin x}{x}$? Write the first four non-zero terms.

21) If $f(x) = x \sin 3x$, what is the Taylor Series for f about $x = 0$? Write the first four non-zero terms.

22) What is the Maclaurin Series for $\frac{1}{(1-x)^2}$? Write the first four non-zero terms.

23 The function f has derivatives of all orders and the Maclaurin series for the function f is given by $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+3}$. Find the Maclaurin series for the derivative $f'(x)$. Write the first four nonzero terms and the general term.

24 If a function has the derivative $f'(x) = \sin(x^2)$ and initial conditions $f(0) = 0$, write the first four nonzero terms of the Maclaurin series for f.

25 What is the sum of the series $1 + \ln 3 + \frac{(\ln 3)^2}{2!} + \cdots + \frac{(\ln 3)^n}{n!}$?

26 Suppose that g is a function which has continuous derivatives, and that $g(5) = 3,\ g'(5) = -2,\ g''(5) = 7,\ g'''(5) = -3$.

(a) What is the Taylor polynomial of degree 3 for g centered at $x = 5$?

(b) Use the polynomial that you found in part (a) to approximate $g(4.9)$.

27 Find a fourth–degree Taylor polynomial for $f(x) = e^{(x-4)}$ centered at $x = 4$.